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Computer Assisted Heuristic Learning of Analysis

Polya's theory has become a wikipedia definition

„The word `Heuristic` means to discover. In this method, the students be put in the place of an independent discoverer. The teacher sets a problem for the students and then ask them to discover the answer. Here students gets only a little help from the teacher. Students find out the solution by experiments, group discussions, or any other means. Experts say that this method has so many positives and it develops scientific attitude. What I would like to earn from you is your valuable view points and experiences about the effectiveness of this method in the college level Mathematics teaching."

How to begin learning calculus?

Rigor in Calculus

- Series and functions
- Continuity
- Limits, finite limits at finite
- Derivative
- Integral

Heuristics method

1. Minima-maxima
2. Area under the curve
   a) Area
   b) Infinity
   c) Functions
   d) Limit at infinity, infinity as a limit
3. Tangent
4. Finding Maxima and Minima using Derivatives

- I. Heuristics method of introduction
- II. After heuristisc introduction we can start to teach the rigor concepts of calculus

I. Heuristics method

Definitions and Explanations

Now, in this construction, there are no definitions for beginners

- There are no incomplete, bad, incomprehensible definitions
- There are no incomprehensible or meaningless proofs

1. Maxima Minima

Knowledge in our Hands

Fixed volume,what about surface?

Extreme value problem for kids: Fixed area, what about the perimeter?

2. Area under the curve

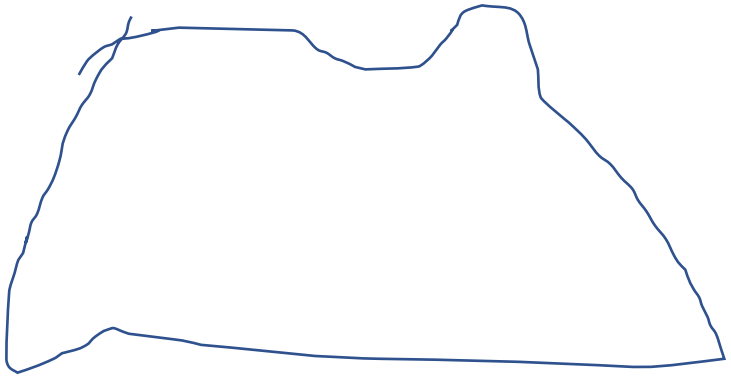
a) Area

We do not ask: What is the meaning of the word „area”? We ask: How to Calculate the Area of a Rectangle?

- An approach to defining what is meant by "area" is through axioms. "Area" can be defined as a function from a collection M of special kind of plane figures (termed measurable sets) to the set of real numbers which satisfies the following properties:
- For all S in M, $a(S) \geq 0$.
- If S and T are in M then so are $S \cup T$ and $S \cap T$, and also $a(S \cup T) = a(S) + a(T) - a(S \cap T)$.
- If S and T are in M with $S \subseteq T$ then $T - S$ is in M and $a(T - S) = a(T) - a(S)$.
- If a set S is in M and S is congruent to T then T is also in M and $a(S) = a(T)$.
- Every rectangle R is in M. If the rectangle has length h and breadth k then $a(R) = hk$.
- Let Q be a set enclosed between two step regions S and T. A step region is formed from a finite union of adjacent rectangles resting on a common base, i.e. $S \subseteq Q \subseteq T$. If there is a unique number c such that $a(S) \leq c \leq a(T)$ for all such step regions S and T, then $a(Q) = c$.
- It can be proved that such an area function actually exists.[11]

Enough Good Solution

- Robbers share the treasure.
- The rule is: one split, another chosen.



Definite integral

Estimate Area, Tutorial:
https://www.youtube.com/watch?v=ohSdtIPrRBk
Estimate area:
https://www.geogebra.org/m/edrchsjs

b) Infinity

We do not ask: What is infinity?

- How many prim number exits?
- If you want it, how can you put a new member, for example pi in the serie of prims?
  (From my students: at the end)
- Hilbert's paradox of the Grand Hotel

c) Functions

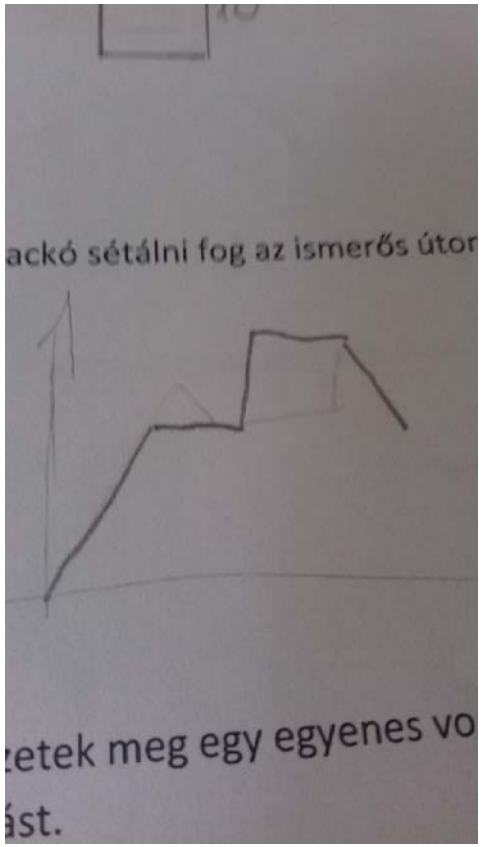
„I am a function"

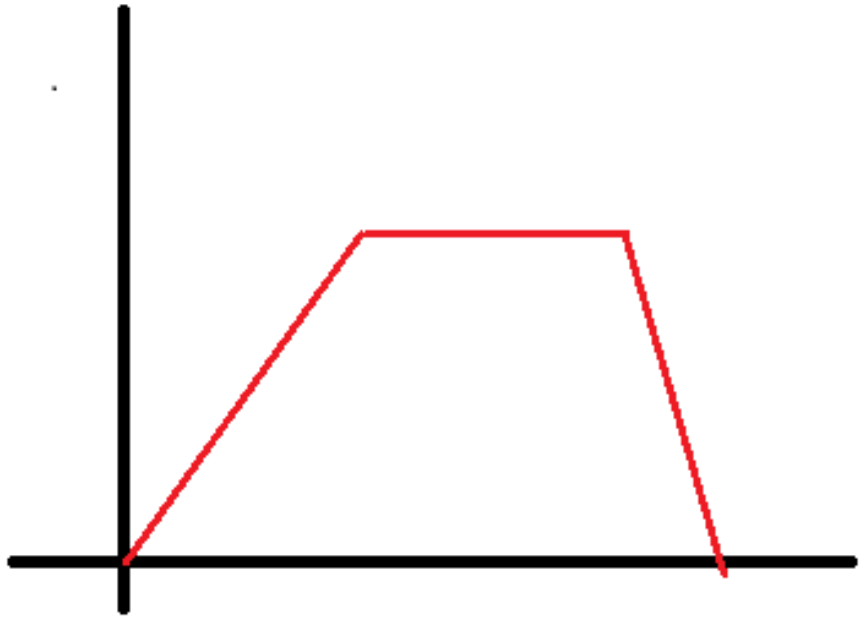
Mathematikum, Giessen, Germany

When we draw a graph with our feet.

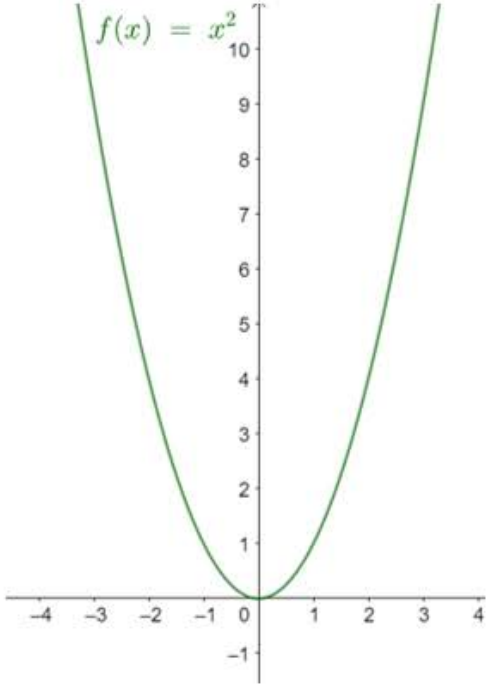
The same in the classroom, without a computer

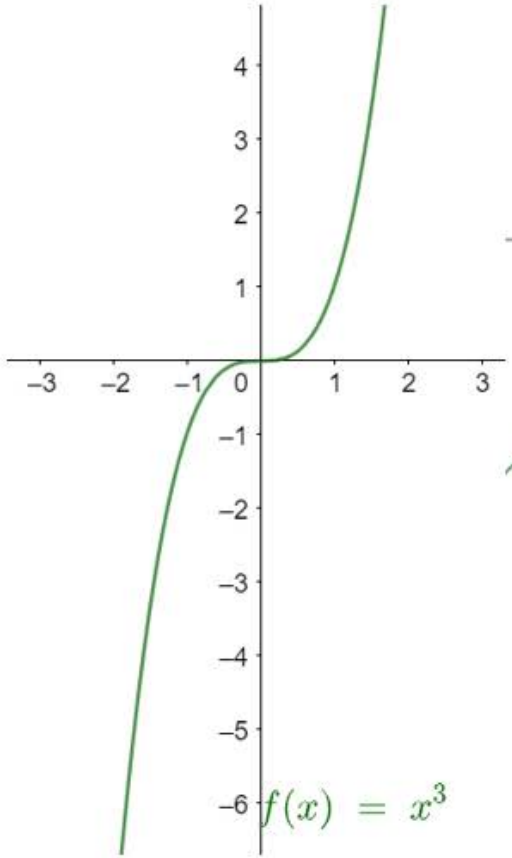
„Write a sheet music about the move!”
Read the graph!

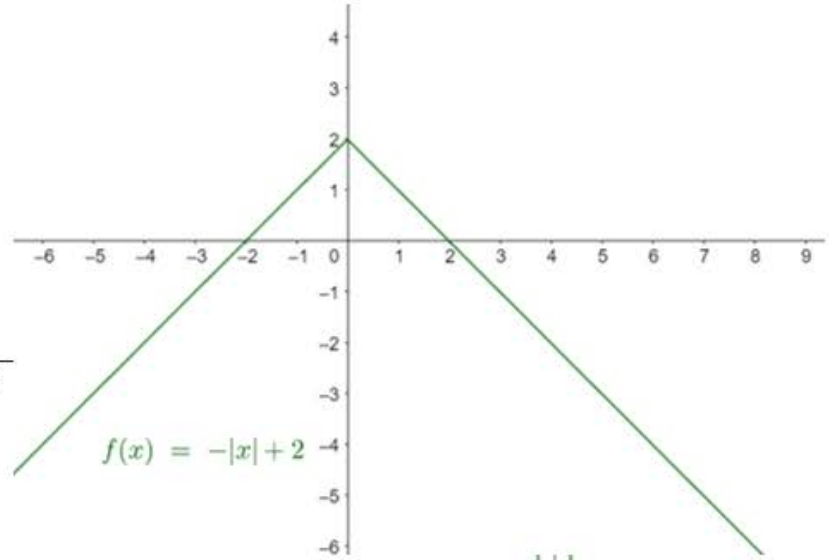


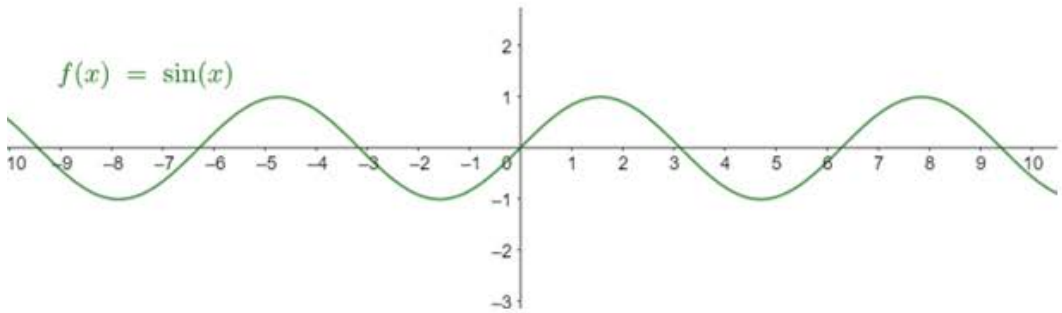


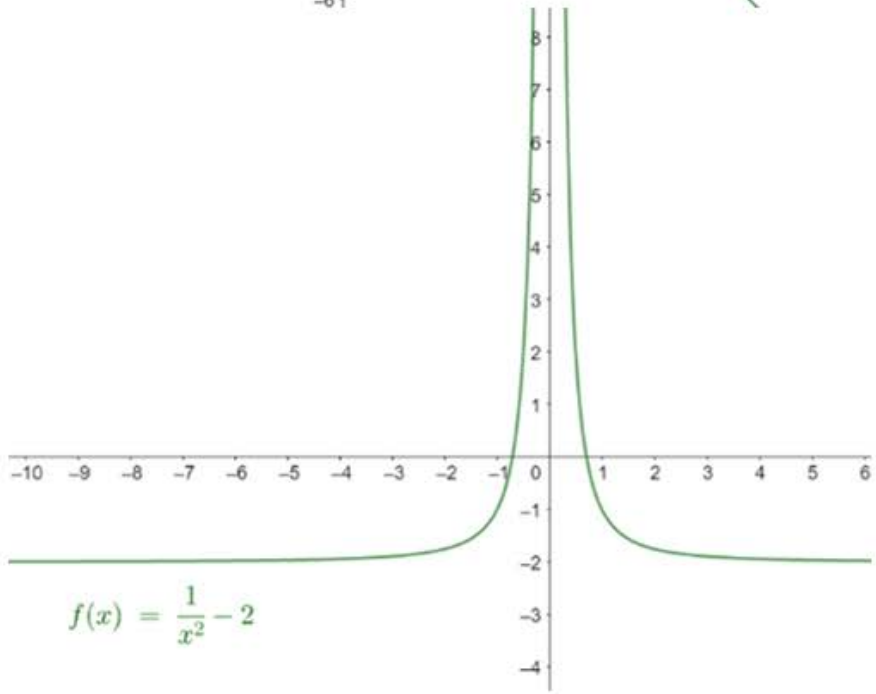
d) Limit at infinity, infinity as a limit


$f(x) = x^2$


$f(x) = x^3$

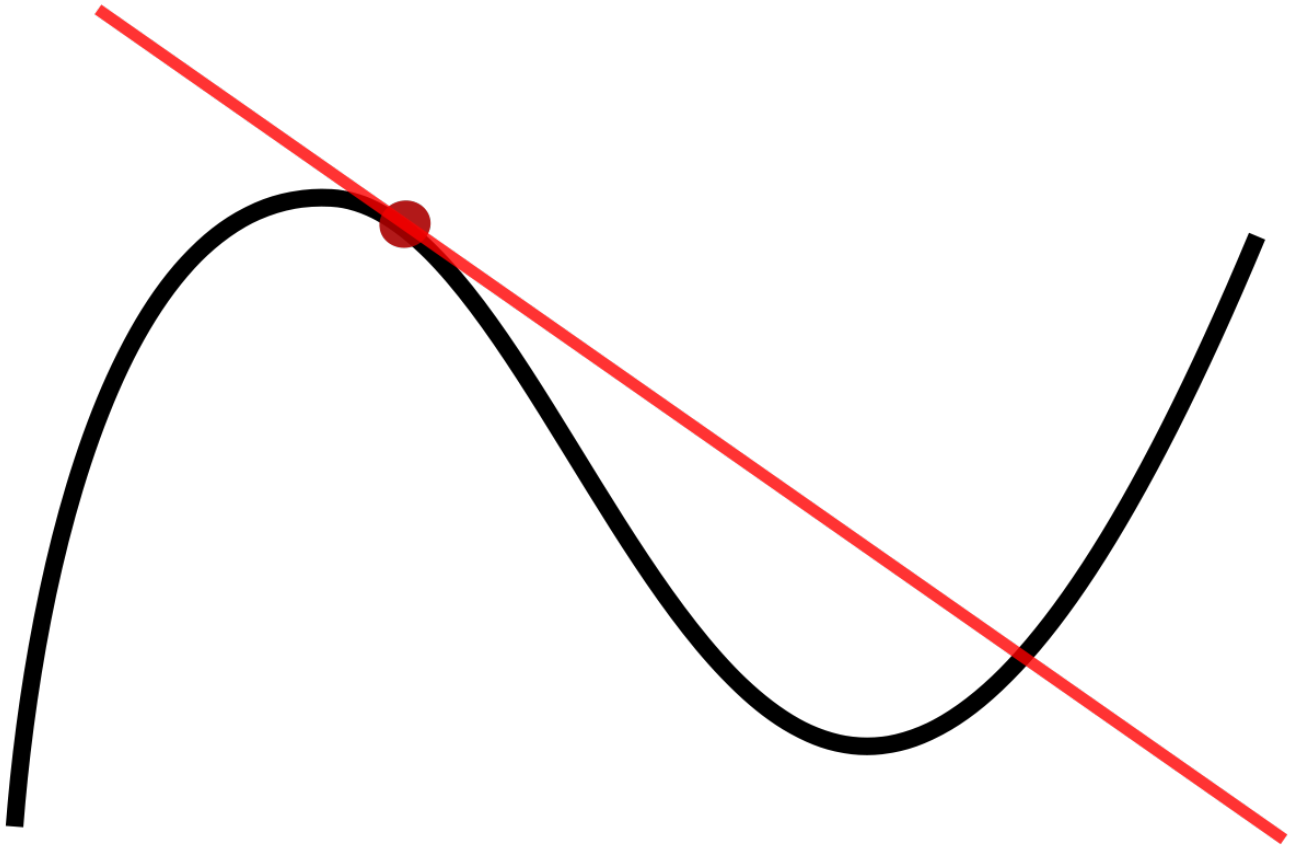

$f(x) = -|x| + 2$


$f(x) = \sin(x)$


$f(x) = \frac{1}{x^2} - 2$

3. Tangent

- A tangent is a straight line that touches a curve at one point. They are both "going in the same direction„.



Give instructions, drawing a tangent

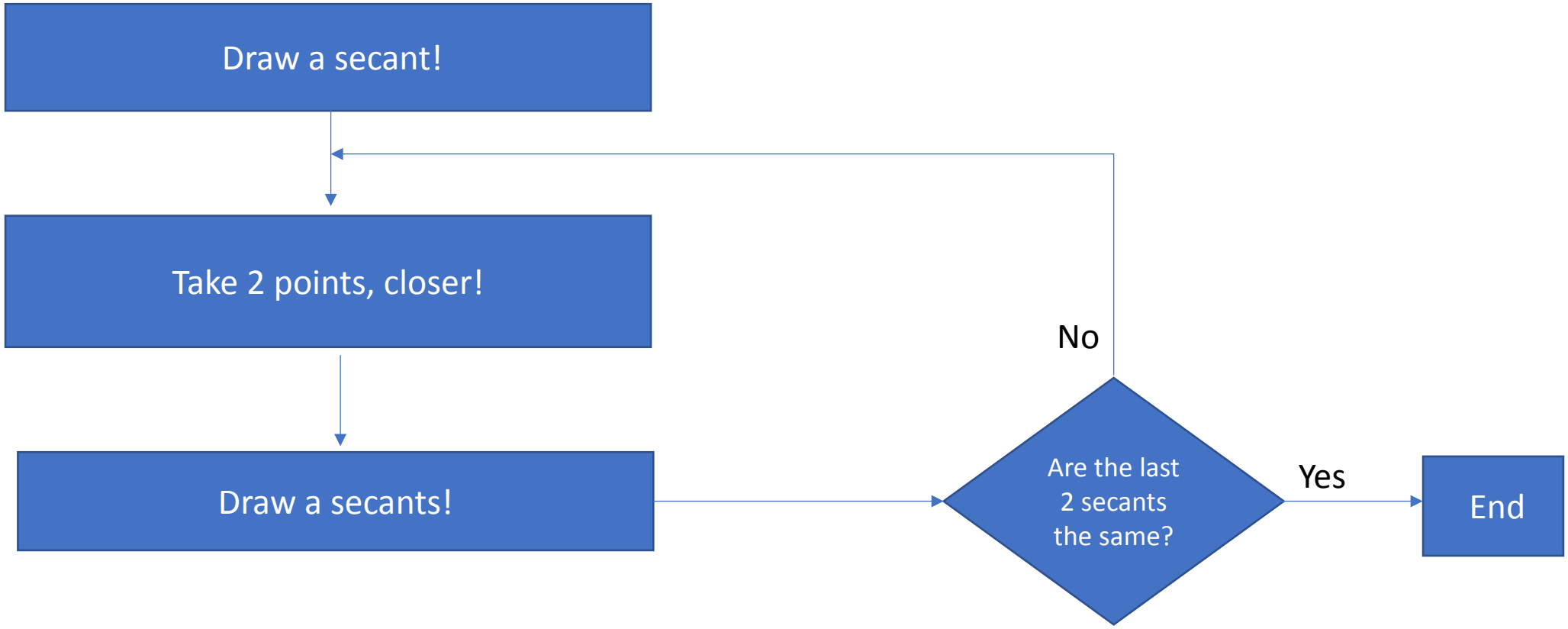
- Instruct your friend to draw a tangent at a given point on a curve. Imagine working in a separate room and keeping in touch over the phone.
- You both know the concept of secant well.

Give instructions, drawing a tangent

- Methodological notes:
- It is preferable to draw a secant belonging to two points equally spaced from A. Students see this better than when only one intersection is moving.
- The next pair of points be at the midpoint of the original distance, because symmetry makes it easier to understand the procedure.
- Instead of the standard introduction of the limit, we chose its equivalent: Cauchy convergence criterion. (The elements of the series converge to each other, not to an existing, but unknown limit).
- Students should try to instruct the teacher to draw a tangent on the board. The teacher turns his back to the students
- The flowchart is a much later task.

Give instructions, drawing a tangent!

Just illustration of our imagination



Estimation of the tangent

- Estimating of the slope, Tutorial: https://www.youtube.com/watch?v=x7hwrNSgD88
- Estimating of the slope: https://www.geogebra.org/m/gtqeredm

The derivative

- The point-by-point derivative of a given function and the derivative function, Tutorial https://www.youtube.com/watch?v=1nUY11FmQ9g
- The point-by-point derivative of a given function and the derivative function: https://www.geogebra.org/m/wvry4ptr

4. Finding Maxima and Minima using Derivatives

- Students can use the derivation rules from the Internet to solve the problems
- At this moment, the derivative rules work like a black box. Based on their experience so far, they have feelings rather than knowledge of them.

II. After heuristisc introduction we can start to teach the rigor concepts of calculus

- Series and functions
- Continuity
- Limits, finite limits at finite
- Derivative
- Integral

We can use heuristics methods also through formal way of learning.

Compute the integral from 0 to x of f(t) for the given functions

- Integralfunction, Manual: https://www.youtube.com/watch?v=9LLpvDeEbP8
- Integralfunction2, Manual: https://www.youtube.com/watch?v=oOQGkPJ3bWk
- Integralfunction2: https://www.geogebra.org/m/aghaxgsj

- Thank you for attention!

Appendix 1

- Érintő meredekség becslése bemutató:
- https://www.youtube.com/watch?v=x7hwrNSgD88
- Érintő meredekség becslése:
- https://www.geogebra.org/m/gtqeredm
- Adott függvény pontonkénti deriváltja és a derivált függvény bemutató:
- https://www.youtube.com/watch?v=1nUY11FmQ9g
- Adott függvény pontonkénti deriváltja és a derivált függvény:
- https://www.geogebra.org/m/wvry4ptr

Appendix 2

- Terület becslése bemutató:
- https://www.youtube.com/watch?v=ohSdtIPrRBk
- Terület becslése:
- https://www.geogebra.org/m/edrchsjs
- Integrálfüggvény1 bemutató:
- https://www.youtube.com/watch?v=9LLpvDeEbP8
- Integrálfüggvény2 bemutató:
- https://www.youtube.com/watch?v=oOQGkPJ3bWk
- Integrálfüggvény2:
- https://www.geogebra.org/m/aghaxgsj